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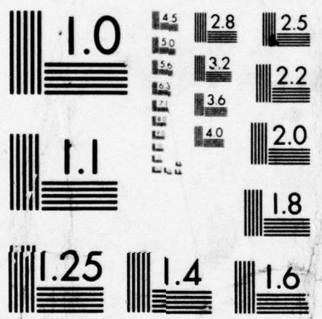

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"SPANWISE-PERIODIC THREE DIMENSIONAL DISTURBANCES IN NOMINALLY 2-D SEPARATING LAMINAR BOUNDARY LAYER FLOWS PART ONE: THEORETICAL FORMULATION" *

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ABSTRACT

This report presents the results of a preliminary investigation on the stability of a nominally-two-dimensional laminar boundary layer flow approaching the separation point with respect to 3-D periodic spanwise disturbances, under the hypothesis that a steady state is reached.

The basic equations of the disturbances are examined with an assumed form of the 3-D perturbations, resulting in a system of ordinary differential equations. Together with the boundary conditions requiring the disturbances to be zero at the wall and to vanish asymptotically at infinity, a two-point eigenvalue problem was formulated. Meksyn's method was adapted for the calculation of the basic two-dimensional flow parameters entering the equations in the region near the separation point. This method, which is superior for practical purposes to Goldstein's well-known analysis of separation, requires the experimentally determined streamwise variation of the pressure at the edge of the boundary layer.

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NOMENCLATURE

a	$= \left(\dfrac{\partial^2 f_o}{\partial \eta^2}\right)_{\eta=0}$; also, speed of sound
D	$= 2\xi(\partial/\partial\xi) - \eta(1 + \Lambda_o + \Sigma_o)(\partial/\partial\eta)$
a_n, b_n, c_n, d_n	coefficients defined in Chapter 3 and Appendixes B and C
$F(\xi,\eta)$	$= \int_o^{\eta} f_o(\xi,\eta)\, d\eta$
$f_o(\xi,\eta)$	transformed stream function; $= \int_o^{y} \dfrac{u_o}{u_e}\, d\eta$
$g_o(\xi,\eta)$	total enthalpy ratio, H_o/H_e
H, h	total and static enthalpy ($H = h + u^2/2$)
ℓ	streamwise characteristic length
p	static pressure
p_n	coefficients defined in Chapter 3 and Appendix B
T	absolute static temperature
u, v, w	streamwise, normal, and sidewash velocity components
x, y, z	streamwise, normal, and lateral coordinate, respectively
α	wave number ($2\pi/\lambda$)
γ	specific heat ratio, c_p/c_v
δ	boundary layer thickness
ξ,η	transformed Levy-Lees coordinates
Λ_o	$= -2\dfrac{d \ln u_e}{d \ln \xi}$
Λ	$= \Lambda_o \dfrac{H_e}{h_e}$;
λ	spanwise disturbance wavelength

μ	laminar coefficient of viscosity
ν	kinematic viscosity (μ/ρ)
ρ	density
τ	shear stress; also, $\tau = \int f_0(\xi,\eta)\, d\eta$

Subscripts

o	basic flow
e	effective edge of the boundary layer
s	separation point
w	wall surface
0, 1, 2,...	order in series expansions

Superscripts

'	disturbance
*	total (or stagnation) values

I. INTRODUCTION

As established by Görtler[1], a two-dimensional flow on a concave wall can develop, under certain conditions, a system of 3-D time-dependent periodic spanwise disturbances featuring a row of alternating streamwise vortex pairs superimposed on the basic two-dimensional boundary-layer (see Figure 1). The concavity of the streamlines and the associated normal pressure variation are the primary mechanisms causing this type of disturbance flow.

A similar type of situation has been found to occur in the case of a two-dimensional flow approaching a stagnation point, which also possesses strongly concave streamlines[2] (Fig. 2). In addition to Görtler's approach a series of papers [2, 3, 4, 5, 6, 7] have demonstrated that this type of disturbance flow can exist even in a steady (i.e., neutrally stable) state. A variety of experimental evidence in support of this is presented in the cited papers.

The facts presented above naturally lead to the idea that these disturbance vortices may in fact occur whenever the basic two-dimensional flow exhibits sufficently concave streamlines. In particular, this might be the case for two-dimensional separating boundary layer flows owing to the significant streamline curvature that develops approaching separation. In fact, a recent experimental investigation by Settles, et al.[9], featuring a detailed examination of the surface flow pattern at separation by the kerosene-graphite technique (see Fig. 3) exhibits a distinctly wavy separation line that suggests the presence of a spanwise-periodic disturbance field. Stimulated by these observations, the present authors therefore undertook a theoretical study of possible vortex-like steady state disturbances near separation.

We consider that the same major qualitative properties of 3-dimensional disturbances in both the flow along a concave wall and in a stagnation region are also valid in the case of a separation region. Consequently, we may assume that the three-dimentional disturbances

(a) are periodic spanwise with a wavelength λ that is larger that the boundary layer thickness (according to Görtler $\lambda \simeq 2\text{-}4\delta$),

(b) are concentrated in a thin layer near the wall whose thickness δ' is not necessarily equal to but of the same order as that of the basic boundary layer,

(c) have a vortex structure featuring a row of streamwise vortex-pairs, and

(d) have a normal variation of the pressure which is a significant aspect of the disturbance mechanism.

In addition, the fact that the separation line is wavy implies that the streamwise disturbance velocity u' cannot be considered zero near the two-dimensional separation point. Unfortunately, the experimental data available until now do not supply any indication about the streamwise variation of disturbances.

However, taking into account the fact that the concavity of the streamlines is likely the main cause of this type of flow and that the concavity has a maximum near the separation line, we can consider that the streamwise variation of disturbances is small enough to be neglected up to the second order.

The model of the flow corresponding to the above mentioned properties is similar with that of Görtler, being represented in Fig. 4.

This report describes the preliminary results of the theoretical investigations on the spanwise disturbed 2-D flow near separation location. In Section 2 we examine the physical basis and assumptions for constructing

a theoretical model followed by a detailed formulation of the appropriate linearized equations and eigenvalue problem governing the small disturbance field. Section 3 discusses at some length the analytical treatment of the basic 2-D separating boundary layer flow itself; we show that this important aspect of the problem is best treated in practice by an adaption of Meksyn's work rather than using Goldstein's well-known mathematical theory of separation. Finally, in Section 4, we outline the suggested future approach to the solution of the equations developed herein.

2. MATHEMATICAL FORMULATION

2.1 Preliminary Appraisal of the Disturbance Mechanism

Prior to a detailed analysis of the disturbance field equations, it is well to first establish the physical plausibility of the hypothesized streamwise vortex disturbance mechanism. This can be done in a very approximate way by a local application of Görtler's instability theory for concave streamline flows; the proposed mechanism is likely to occur if under typical conditions it appreciably exceeds his instability criterion.

According to Görtler[1], a concave boundary layer flow having an average longitudinal radius of curvature R will be susceptible to amplified streamwise vortex disturbances whenever the following instability condition is met:

$$\frac{\Theta^* U_e}{\nu}\sqrt{\frac{\Theta^*}{R}} = (\Theta^*/L)^{3/2}\left(\frac{Re_L}{\sqrt{R/L}}\right) \gtrsim .25 \qquad (1)$$

where Θ^* is the momentum thickness and L is the streamwise distance to the local curvature region under consideration. Now to apply Eq. (1) to the present problem$^+$, we must supply an estimate of R typical of the boundary layer flow near the surface approaching separation. This can be found from the following analytical expression (derived in Appendix A-1) that relates R to the observable physical properties of the flow (see Fig. 4):

$$R \approx (\partial p/\partial x)_s / (\partial^2 \tau_w / \partial x^2)_s \qquad (2)$$

$^+$ Since the boundary layer profile upon which Eq.(1)is based has a significant wall shear, its application near separation is only a rough approximation because it is not known how the RHS is influenced by an adverse pressure gradient.

We thus require data on the streamwise pressure gradient and <u>second</u> derivative of the streamwise wall shear distribution [i.e., the curvature of the graph $\tau_w(x)$] at the separation point.

Fortunately, there exists some relatively detailed experimental data at conditions close to those pertaining to Fig. 3 from which these quantities can be readily extracted: Figures 5 & 6 illustrate the pressure and wall shear distributions measured by Sfeir for a $M_\infty = 2.64$, $Re_L = 1.4 \times 10^5$ laminar flow separating in front of a small angle ramp compression corner. From these curves, we obtain the estimates

$$[\partial^2 \tau_w / \partial(X/L)^2]_s \simeq .4\ \tau_{w,ref}$$

and

$$[\partial(p/p_\infty)/\partial(X/L)]_s \simeq 1.7$$

where $\tau_{w,ref} \simeq .332\ \rho_e U_e^2 / \sqrt{Re_x}$ is the Blasius reference value at $X/L \simeq .25$ with $L \simeq X_s \simeq 2.7$ inches and $\Theta_s^* \simeq .07$ inches. Consequently we find by substitution into Eq.(2) that

$$\frac{R}{L} \simeq 2.4 \frac{\sqrt{Re_L}}{M_\infty^2} \simeq 129 \tag{3}$$

which in turn from Eq.(1) yields the stability parameter value

$$(\Theta^* U_\infty / \nu) \sqrt{\Theta^*/R} \simeq 28.6. \tag{4}$$

Since this exceeds the critical value by two orders of magnitude, it is likely that the streamline curvature approaching separation in these typical conditions is more than sufficient to give birth to streamwise vortex-like disturbances.

2.2 Basic Assumptions, Order of Magnitude Analysis and General Disturbance Equations

In order to derive the basic disturbance equations, we start from the

complete Navier Stokes equations of viscous compressible flow and as a first step, make use of the following assumptions.

1. The flow is composed of a basic two-dimensional flow u_0, v_0, p_0, ρ_0, T_0 plus a system of three-dimensional steady state perturbations u', v', w', p', ρ', T'. We note in this connection that the basic flow is not assumed to be a parallel shear flow (indeed, in the present study such an assumption would eliminate the disturbances of interest).

2. The perturbations are small, so that we can neglect second and higher order terms as compared with those of first order (linearization).

3. Provided that the Reynolds number is sufficiently large, the boundary layer-like approximations apply to the basic flow parameters, these furnishing the well-known estimations that

$$\frac{\delta_0}{\ell_0} \simeq \frac{1}{\sqrt{Re}} = \sqrt{\frac{\mu_0}{\rho_0 U_\infty \ell_0}} << 1,$$

$$v_0 \simeq \frac{\delta_0}{\ell_0} u_0 \simeq \frac{\delta_0}{\ell_0} U_\infty,$$

$$\frac{\partial}{\partial x} << \frac{\partial}{\partial y} .$$

4. Under the same condition, the viscous effects on the disturbances are concentrated in a thin layer whose thickness δ' is considered of the same order as δ_0 and small as compared with both the spanwise and streamwise characteristic lengths. Taking into account that Görtler's value for the spanwise wavelength is $\lambda \simeq (2 - 4)\ \delta_0$, we assume $\delta'^2/\lambda^2 << 1$ and thus

$$\frac{\partial^2}{\partial z^2} << \frac{\partial^2}{\partial y^2} .$$

As for the streamwise characteristic length, we will consider $\delta'/\ell' << 1$ and hence $\partial/\partial x << \partial/\partial y$ (at the same time we must also have $\lambda/\ell' \lesssim \delta'/\lambda$, $\lambda^2/\ell'^2 << 1$).

5. As in Gortler's flow along concave walls, the streamwise disturbance vorticity ω'_x dominates the normal disturbance vorticity ω'_y. This can be expressed by

$$\left| \frac{\partial u'}{\partial z} - \frac{\partial w'}{\partial x} \right| < \left| \frac{\partial w'}{\partial y} - \frac{\partial v}{\partial z} \right| \tag{5}$$

6. Considering that the compressibility does not change essentially the proportions between u', v', and w', we obtain the following estimation by using the incompressible disturbance continuity equation:

$$v' < \delta' \left|\frac{\partial w'}{\partial z}\right| + \delta' \left|\frac{\partial u'}{\partial x}\right| \tag{6}$$

On the other hand,

$$\delta' \left|\frac{\partial^2 u'}{\partial z \partial x}\right| \ll \left|\frac{\partial u'}{\partial z}\right| , \quad \delta' \left|\frac{\partial^2 w'}{\partial z^2}\right| \ll \left|\frac{\partial w'}{\partial y}\right| , \text{ and } \frac{\partial w'}{\partial x} << \frac{\partial w'}{\partial y} .$$

Consequently, Eq. (5) gives:

$$\left|\frac{\partial u'}{\partial z}\right| < \left|\frac{\partial w'}{\partial y}\right|, \quad u' < \frac{\lambda}{\delta'} w' \tag{7}$$

and hence from Eq. (6)

$$v' \lesssim \frac{\delta'}{\lambda} w' \tag{8}$$

Applying the above assumptions and assuming also a unit Prandtl number and Chapmann's law for the laminar viscosity (μ/T = const.), the following general disturbance equations are obtained:

Continuity

$$\frac{\partial}{\partial x}(\rho_o u' + u_o \rho') + \frac{\partial}{\partial y}(\rho_o v' + v_o \rho') + \rho_o \frac{\partial w'}{\partial z} = 0 \tag{9}$$

Streamwise momentum

$$\rho' \left(u_o \frac{\partial u_o}{\partial x} + v_o \frac{\partial u_o}{\partial y}\right) + \rho_o u' \frac{\partial u_o}{\partial x} + \rho_o u_o \frac{\partial u'}{\partial x} + \rho_o v' \frac{\partial u_o}{\partial y} + \rho_o v_o \frac{\partial u'}{\partial y} + \frac{\partial p'}{\partial x}$$

$$- \frac{\partial}{\partial y}\left(\mu' \frac{\partial u_o}{\partial y}\right) - \frac{\partial}{\partial y}\left(\mu_o \frac{\partial u'}{\partial y}\right) \simeq 0 \tag{10}$$

Normal Momentum

$$\rho'(u_o \frac{\partial v_o}{\partial x} + v_o \frac{\partial v_o}{\partial y}) + \rho_o u' \frac{\partial v_o}{\partial x} + \rho_o u_o \frac{\partial v'}{\partial x} + \rho_o v' \frac{\partial v_o}{\partial y} + \rho_o v_o \frac{\partial v'}{\partial y}$$

$$- \frac{\partial}{\partial z}(\mu_o \frac{\partial w'}{\partial y}) - \frac{\partial}{\partial x}(\mu' \frac{\partial u_o}{\partial y}) - \frac{\partial}{\partial x}(\mu_o \frac{\partial u'}{\partial y}) - \frac{2}{3}\frac{\partial}{\partial y}[\mu'(2\frac{\partial v_o}{\partial y} - \frac{\partial u_o}{\partial x})]$$

$$- \frac{2}{3}\frac{\partial}{\partial y}[\mu_o(2\frac{\partial v'}{\partial y} - \frac{\partial u'}{\partial x} - \frac{\partial w'}{\partial z})] \simeq - \frac{\partial p'}{\partial y} \quad (11)$$

Spanwise Momentum

$$\rho_o u_o \frac{\partial w'}{\partial x} + \rho_o v_o \frac{\partial w'}{\partial y} - \frac{\partial}{\partial y}(\mu_o \frac{\partial w'}{\partial y}) \simeq - \frac{\partial p'}{\partial z} \quad (12)$$

Energy

$$\rho' (u_o \frac{\partial H_o}{\partial x} + v_o \frac{\partial H_o}{\partial y}) + \rho_o u_o \frac{\partial H'}{\partial x} + \rho_o u' \frac{\partial H_o}{\partial x} + \rho_o v_o \frac{\partial H'}{\partial y}$$

$$+ \rho_o v' \frac{\partial H_o}{\partial y} \simeq \frac{\partial}{\partial y}(\mu_o \frac{\partial H'}{\partial y}) + \frac{\partial}{\partial y}(\mu' \frac{\partial H_o}{\partial y}) \quad (13)$$

Equation of State

$$\frac{\rho'}{\rho_o} = \frac{p'}{p_o} - \frac{T'}{T_o} \quad (14)$$

Chapmann's Law

$$\frac{\mu'}{\mu_o} = \frac{T'}{T_o} \quad (15)$$

Enthalpy

$$H' = c_p T' + u_o u' \quad (16)$$

It is noted that at this point the disturbance field may be non-adiabatic.

It was mentioned in the Introduction that the normal disturbance pressure variation is an important mechanism of the lateral stability of the flow. This rises from the fact that the normal and spanwise momentum equations are strongly coupled through the pressure gradient terms even though $\partial p/\partial y$ may be small. However, at the same time we <u>can</u> neglect the

effect of the pressure disturbance in the state equation. To see this, consider the normal momentum equation estimating the orders of magnitude of all entering terms. Upon integration with respect with y, the following result is obtained for the order of magnitude of the pressure:

$$\frac{p'}{p_o} \simeq \text{terms of order}\left(\frac{u_o^2}{RT_o}\,\frac{\delta_o^2}{\ell_o^2}\,\frac{u'}{u_o}\right) + \text{terms of order}\left(\frac{u_o^2}{RT_o}\,\frac{\delta_o}{\ell_o}\,\frac{v'}{u_o}\right)$$

$$+ \text{terms of order}\left(\frac{u_o^2}{RT_o}\,\frac{\delta_o^2}{\ell_o\lambda}\,\frac{w'}{u_o}\right) + \text{terms of order}\left(\frac{u_o^2}{RT_o}\,\frac{\delta_o^2}{\ell_o^2}\,\frac{\rho'}{\rho_o}\right)$$

$$+ \text{terms of order}\left(\frac{u_o^2}{RT_o}\,\frac{\delta_o^2}{\ell_o^2}\,\frac{T'}{T_o}\right) \qquad (17)$$

At the same time, from the relation (16) we obtain

$$\frac{T'}{T_o} \simeq \frac{H'}{C_pT_o} - \left(\frac{\gamma-1}{2\gamma}\right)\frac{u_o^2}{RT_o}\,\frac{u'}{u_o}$$

Comparing p'/p_o and T'/T_o (to simplify, we consider the adiabatic case $H' = 0$ without loss of generality), it is seen that when the Reynolds number is large ($\delta_o/\ell_o \ll 1$) either $p'/p_o \ll T'/T_o$ or, if $u' \simeq 0$, both p'/p_o and T'/T_o can be neglected and hence $\rho'/\rho_o \simeq 0$. Both cases are included in the simplified state equation

$$\frac{\rho'}{\rho_o} \simeq \frac{T'}{T_o} \qquad (14')$$

We note that this simplification cannot be used in the region of the stagnation point where $\delta_o/\ell_o > 1$.

The pressure can be eliminated between the normal and spanwise momentum by cross derivation and subtraction. As a result we obtain the equation

$$\frac{\partial\rho'}{\partial z}\left(u_o\frac{\partial v_o}{\partial x} + v_o\frac{\partial v_o}{\partial y}\right) + \rho_o\frac{\partial v_o}{\partial x}\frac{\partial u'}{\partial z} - \rho_o u_o\frac{\partial^2 w'}{\partial y\partial x} - \frac{\partial\rho_o u_o}{\partial y}\frac{\partial w'}{\partial x} - \rho_o v_o\frac{\partial^2 w'}{\partial y^2}$$

$$- \frac{\partial \rho_o v_o}{\partial y} \frac{\partial w'}{\partial y} + \frac{\partial^2}{\partial y^2}(\mu_o \frac{\partial w'}{\partial y}) \quad + \quad \rho_o u_o \frac{\partial^2 v'}{\partial z \partial x} + \rho_o \frac{\partial v_o}{\partial y} \frac{\partial v'}{\partial z} + \rho_o v_o \frac{\partial^2 v'}{\partial z \partial y} \quad =$$

$$+ \mu_o \frac{\partial^3 w'}{\partial z^2 \partial y} + \frac{\partial^2}{\partial z \partial x} (\mu' \frac{\partial u_o}{\partial y}) + \frac{\partial^2}{\partial z \partial x} (\mu_o \frac{\partial u'}{\partial y}) + \frac{2}{3} \frac{\partial^2}{\partial z \partial y} [\mu'(2 \frac{\partial v_o}{\partial y} - \frac{\partial u_o}{\partial x})]$$

$$+ \frac{2}{3} \frac{\partial^2}{\partial z \partial y} [\mu_o(2 \frac{\partial v'}{\partial y} - \frac{\partial u'}{\partial x} - \frac{\partial w'}{\partial z})] \tag{18}$$

We can see now that upon the assumption $\delta'^2/\lambda^2 << 1$ used together with the boundary layer approximations, the terms in the right hand side of the above equation can be neglected. As a consequence we obtain finally

$$\frac{\partial \rho'}{\partial z} (u_o \frac{\partial v_o}{\partial x} + v_o \frac{\partial v_o}{\partial y}) + \rho_o \frac{\partial v_o}{\partial x} \frac{\partial u'}{\partial z} - \rho_o u_o \frac{\partial^2 w'}{\partial y \partial x} - \frac{\partial \rho_o u_o}{\partial y} \frac{\partial w'}{\partial x} - \rho_o v_o \frac{\partial^2 w'}{\partial y^2} +$$

$$+ \rho_o \frac{\partial}{\partial y} (v_o \frac{\partial v'}{\partial z}) - \frac{\partial \rho_o v_o}{\partial y} \frac{\partial w'}{\partial y} + \frac{\partial^2}{\partial y^2} (\mu_o \frac{\partial w'}{\partial y}) + \rho_o u_o \frac{\partial^2 v'}{\partial z \partial x} \simeq 0 \tag{18'}$$

2.3 Assumed Form of Disturbances and Resulting Simplified Equations

The equations established above are generally valid irrespective of the specific spanwise or streamwise variation of the disturbances. Now we specialize them to the spanwise periodic vortex-like disturbance field mentioned previously by postulating the following forms for u', v', p', ρ', T' near the separation point:

$$\begin{aligned} u' &= u_s'(y) \cos \alpha z + O(x-x_s)^2 \\ v' &= v_s'(y) \cos \alpha z + O(x-x_s)^2 \\ w' &= w_s'(y) \sin \alpha z + O(x-x_s)^2 \\ \rho' &= \rho_s'(y) \cos \alpha z + O(x-x_s)^2 \\ p' &= p_s'(y) \cos \alpha z + O(x-x_s)^2 \\ T' &= T_s'(y) \cos \alpha z + O(x-x_s)^2 \end{aligned} \tag{19}$$

where $\alpha = 2\pi/\lambda$ is the spanwise wave number and the subscript s refers to the nominal 2-D separation point. These formulas include also the requirement that the perturbations undergo a streamwise maximum near the separation point. A schematic illustrating the associated flow pattern is shown in Fig. 1.

The assumed solution (15) reduces the number of independent variables from three to one, thus converting the partial differential equations into a set of ordinary differential equations governing the normal variation of the disturbances. At the same time the streamwise gradient of the pressure disturbance is eliminated from the streamwise momentum equation. Using the equations (14'), (15), and (16) the density, viscosity, and enthalpy disturbances can also be eliminated. Consequently we ultimately obtain the following set of equations for u_s', v_s', w_s', and T_s':

$$\rho_o \frac{\partial v_s'}{\partial Y} - \rho_o v_o \frac{\partial (T_s'/T_o)}{\partial Y} + \frac{1}{\rho_o}\frac{\partial \rho_o}{\partial x} u_s' - \left(\frac{\partial u_o}{\partial x} + \frac{\partial \rho_o v_o}{\partial Y} \right) \frac{T_s'}{T}$$

$$+ \frac{\partial \rho_o}{\partial Y} v_s' + \alpha\, w_s' \simeq 0 \qquad (20)$$

$$\frac{\partial u_o}{\partial x} u_s' + \frac{\partial u_o}{\partial y} v_s' + \rho_o v_o \frac{\partial u_s'}{\partial Y} \simeq \left(u_o \frac{\partial u_o}{\partial x} + v_o \frac{\partial u_o}{\partial y} + \rho_o \mu_o \frac{\partial^2 u_o}{\partial Y^2} \right) \frac{T_s'}{T_o}$$

$$+ \rho_o \mu_o \frac{\partial u_o}{\partial Y} \frac{\partial (T_s'/T_o)}{\partial Y} + \rho_o \mu_o \frac{\partial^2 u_s'}{\partial Y^2} \qquad (21)$$

$$- \frac{\partial v_o}{\partial x} \alpha u_s' - \rho_o^{\,2} v_o \frac{\partial^2 w_s'}{\partial Y^2} - \frac{\partial \rho_o v_o}{\partial y} \frac{\partial w_s'}{\partial Y} + \rho_o^{\,2} \mu_o \frac{\partial^3 w_s'}{\partial Y^3}$$

$$- \left(u_o \frac{\partial v_o}{\partial x} + \rho_o v_o \frac{\partial v_o}{\partial Y} \right) \frac{T_s'}{T_o} \qquad (22)$$

$$\frac{\partial H_o}{\partial x} u_s' + \rho_o v_o \frac{\partial[h_o (T_s'/T_o) + u_o u_s']}{\partial Y} + \rho_o \frac{\partial H_o}{\partial Y} v_s' \simeq \left(u_o \frac{\partial H_o}{\partial x} + \rho_o v_o \frac{\partial H_o}{\partial Y}\right.$$

$$\left. + \rho_o \mu_o \frac{\partial^2 H_o}{\partial y^2}\right) \frac{T_s'}{T_o} + \rho_o \mu_o \frac{\partial H_o}{\partial Y} \frac{\partial (T_s'/T_o)}{\partial Y} +$$

$$+ \rho_o \mu_o \frac{\partial^2 [h_o (T_s'/T_o) + u_o u_s']}{\partial Y^2} \tag{23}$$

where $dY = \rho_o dy$ and $h_o = c_p T_o$.

We note that when the basic flow is adiabatic (H_o = const.) the equation (23) admits the solution $H' = h_o(T_s'/T_o) + u_o u_s' = 0$ which means that the 3-D disturbed flow is also adiabatic (insulating wall).

2.4 Disturbance Equations in Terms of Levy-Lees Transformed Coordinates

As will be shown in the next chapter, Levy-Lees transformed coordinates (including both Howarth-Dorodnitsyn and Mangler transformations) are more convenient for the description of the basic two-dimensional flow near the separation point and hence for the expression of the coefficients entering the disturbance equations.

In terms of Levy-Lees transformed coordinates

$$\xi = \int_0^x \rho_w \mu_w u_e^2 dx \quad , \quad \eta = \frac{u_e \int_0^y \rho_o dy}{2\xi} \tag{24}$$

The basic two-dimensional flow is described by the well known incompressible-like equations [14] :

$$\frac{\partial^3 f_o}{\partial \eta^3} + f_o \frac{\partial^2 f_o}{\partial \eta^2} = \Lambda[g_o - (\frac{\partial f_o}{\partial \eta})^2] + 2\xi(\frac{\partial f_o}{\partial \eta}\frac{\partial^2 f_o}{\partial \eta \partial \xi} - \frac{\partial f_o}{\partial \xi}\frac{\partial^2 f_o}{\partial \eta^2}) \tag{25}$$

$$\frac{\partial^2 g_o}{\partial \eta^2} + f_o \frac{\partial g_o}{\partial \eta} = 2\xi\ (\frac{\partial f_o}{\partial \eta}\frac{\partial g_o}{\partial \xi} - \frac{\partial f_o}{\partial \xi}\frac{\partial g_o}{\partial \eta}) \tag{26}$$

where $\partial f_o/\partial \eta = u_o/u_e$, $g_o = H_o/H_e$ and

$$\Lambda = -2\frac{d\ \ln u_e}{d\ \ln \xi}\ \frac{H_e}{h_e} = -\frac{2\xi}{\rho_w \mu_w u_e{}^2}\ \frac{du_e}{dx}\ \frac{H_e}{h_e} \tag{27}$$

Using the new coordinates and notations, the basic flow parameters can be expressed by the following relations

$$\rho_o v_o = -\frac{\rho_w \mu_w u_e}{\sqrt{2\xi}}\left[2\xi\frac{df_o}{d\xi} + f_o - \eta(1 + \Lambda_o + \Sigma_o)\frac{df_o}{d\eta}\right] \tag{28}$$

$$T_o = T_e{}^*\left[g_o - \frac{u_e{}^2}{2c_p T_e{}^*}\left(\frac{df_o}{d\eta}\right)^2\right] = T_e{}^*\left[g_o - \chi\left(\frac{df_o}{d\eta}\right)^2\right] \tag{29}$$

where $$\Lambda_o = \Lambda\frac{h_e}{H_e} = -2\frac{h_e}{H_e}\frac{d\ \ln u_e}{d\ \ln \xi}\ ,\ \Sigma_o = -2\frac{d\ \ln \int_0^y \rho\ dy}{d\ \ln \xi} \tag{30}$$

$$\chi = \frac{u_e{}^2}{2c_p T_e{}^*} = \frac{\gamma-1}{2} M_e{}^2\frac{h_e}{H_e} \tag{31}$$

Note that the flow outside the boundary layer is isentropic so we have

$$H_e = H_\infty, \qquad T_e{}^* = T_\infty{}^* \tag{32}$$

$$\frac{h_e}{H_e} = \frac{T_e}{T_e{}^*} = \frac{T_\infty}{T_\infty{}^*}\left(\frac{P_e}{P_\infty}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_\infty}{T_\infty{}^*}\left(C_{pe}\right)^{\frac{\gamma-1}{\gamma}} \tag{33}$$

$$u_e\frac{du_e}{dx} = -RT_e\frac{1}{C_{pe}}\frac{dC_{pe}}{dx} \tag{34}$$

where $C_{pe} = p_e/p_\infty$ and R is the gas constant.

We assume at this point that the functions f_o and g_o are regular at the separation location* so we can write

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* This problem will be discussed in more detail in the next chapter.

$$f = f_{os} + \left(\frac{\partial f_o}{\partial \xi}\right)_s (\xi - \xi_s) + \left(\frac{\partial^2 f_o}{\partial \xi^2}\right)_s (\xi - \xi_s)^2 + \ldots$$

$$g = g_{os} + \left(\frac{\partial g_o}{\partial \xi}\right)_s (\xi - \xi_s) + \left(\frac{\partial^2 g_o}{\partial \xi^2}\right)_s (\xi - \xi_s)^2 + \ldots \qquad (35)$$

Introduce now the following adimensional quantities instead of u_s', v_s', w_s', T_s' and α

$$U = \frac{u_s'}{u_{es}}$$

$$V = \frac{\sqrt{2\xi_s}}{\mu_{ws}} \frac{v_s'}{u_{es}}$$

$$W = \frac{1}{\bar{\alpha}} \frac{\sqrt{2\xi_s}}{\mu_{ws}} \frac{w_s'}{u_{es}} \qquad (36)$$

$$T = \frac{T_s'}{T_o}$$

$$\bar{\alpha} = \frac{\sqrt{2\xi_s}}{\rho_w u_{es}} \alpha$$

Confining our attention to a small neighbourhood of the separation point and employing also the differential transformations resulting from (24)

$$\frac{\partial \cdot}{\partial x} = \rho_w \mu_w u_e \left[\frac{\partial \cdot}{\partial \xi} - \frac{\eta}{2\xi}(1 + \Lambda_o + \Sigma_o)\frac{\partial \cdot}{\partial \eta}\right] = \frac{\rho_w \mu_w u_e}{2\xi} D \cdot \qquad (37)$$

$$\frac{\partial}{\partial y} = \frac{\rho_o u_e}{\sqrt{2\xi}} \frac{\partial}{\partial \eta} \qquad (38)$$

The disturbance equations (20), (21), (22), and (23) are written in terms of the above notations and Levy-Lees transformed coordinates

$$\bar{\alpha}^2 W + \frac{\partial}{\partial \eta}(F_{gs}V) + [\gamma M_{es}{}^2 \Lambda_{os} - D_s(F_g/g_w)]\,U + [D_s(f_o) + f_{os}]\,\frac{\partial T}{\partial \eta}$$

$$- \Sigma_{os} T = 0 \tag{39}$$

$$\frac{\partial^2 U}{\partial \eta^2} + [D_s(f_o) + f_{os}]\,\frac{\partial U}{\partial \eta} + [\Lambda_{os}\frac{\partial f_{os}}{\partial \eta} - D_s\left(\frac{\partial f_o}{\partial \eta}\right)]\,U - F_{gs}\frac{\partial^2 f_{os}}{\partial \eta^2}\,V$$

$$+ (2\,\frac{\partial^2 f_{os}}{\partial \eta^2} - \Lambda_{os})\,T + \frac{\partial f_{os}}{\partial \eta}\,\frac{\partial T}{\partial \eta} = 0 \tag{40}$$

$$- [F_{os}(1 + \Lambda_{os}) - D_s(F_o)]\,U + F_{gs}{}^2\,\frac{\partial F_{os}V}{\partial \eta} + F_{os}F_{gs}{}^2\,\frac{\partial^2 W}{\partial \eta^2}$$

$$+ F_{gs}\frac{\partial}{\partial \eta}[(D_s(f_o) + f_{os})\,\frac{\partial W}{\partial \eta}] + F_{gs}\frac{\partial^3 W}{\partial \eta^3} - \left\{ \frac{\partial f_{os}}{\partial \eta}[F_{os}(1 + \right.$$

$$\left. \Lambda_{os}) - D_s(F_o)] + \frac{\partial F_{os}}{\partial \eta}[f_{os} + D_s(f_o)] \right\} T = 0 \tag{41}$$

$$D_s(g_o)\,U + [f_{os} + D_s(f_o)]\,\frac{\partial[(g_{ws}/F_{gs}) + 2\,{}_sU]}{\partial \eta} + F_{gs}\frac{\partial g_{os}}{\partial \eta}\,V$$

$$\simeq 2\,\frac{\partial^2 g_{os}}{\partial \eta^2}\,T + \frac{\partial g_{os}}{\partial \eta}\,\frac{\partial T}{\partial \eta} + \frac{\partial[(g_{ws}/F_{gs}) + 2\,{}_sU]}{\partial \eta} \tag{42}$$

where for convenience we have introduced the notations

$$F_o = \frac{1}{g_{ws}}\left[g_o - \chi\left(\frac{\partial f_o}{\partial \eta}\right)^2\right]\left[2\xi\frac{\partial f_o}{\partial \xi} + f_o - \eta\,(1 + \Lambda_o + \Sigma_o)\,\frac{\partial f_o}{\partial \eta}\right] \tag{43}$$

$$F_g = \frac{g_w}{g_o - \chi\left(\frac{\partial f_o}{\partial \eta}\right)^2} \tag{44}$$

2.5 Boundary Conditions for Disturbances and Resulting Eigenvalue Problem

The disturbance equations must be accompanied by the appropriate boundary conditions at the wall and far from the wall. The physical conditions of no-slip and zero mass transfer (blowing or suction) require that $U_s'(0) = V_s'(0) = W_s'(0) = 0$ at the wall. As for the temperature, the problem is much more complicated by the "spanwise thermal response" (see for example [7]) of the solid body to the thermal disturbances in the boundary layer. In the present work we shall assume that the surface remains at the same given temperature τ_w. Consequently, we put $T_s'(0) = 0$. On the other hand, far from the wall and well outside the boundary layer we assume that the disturbances vanish asymptotically and we put

$$U_s'(\infty) = V_s'(\infty) = W_s'(\infty) = T_s'(\infty) = 0$$

In practice, these conditions are implemented by constructing inviscid asymptotic solutions (far outside the boundary layer) which satisfy them, and then imposing a numerical matching of the inner solution with the asymptotic values at some finite but large value of η.

The set of differential equations (39) - (42) along with the aforementioned boundary conditions, when supplemented by the solution for the basic 2-D separating boundary layer profile functions and associated pressure gradient parameters for a given wall temperature, constitute a well-posed (if rather complicated) eigenvalue problem for the assumed spanwise disturbance vortices. Since the basic flow possesses a definite physical scale (namely the boundary layer thickness δ_0),we might expect to find a discrete set of eigenvalues $\bar{\alpha}$ with a dominant fundamental wavelength $\lambda \sim O(\delta_0)$. This similar to Görtler's problem but not the reattachment disturbance problem where a continuous spectrum of eigenvalues was obtained in an infinitely-wide incoming flow.

3. BASIC UNDISTURBED 2-D FLOW DESCRIPTION NEAR A SEPARATION POINT (ADIABATIC CASE)

3.1 General Considerations

The problem of separation of the laminar boundary layer has been considered from the beginning as one of the most important for the prediction of viscous flows. The main difficulty arises from the fact that separation is the result of the interaction between the viscous flow in the boundary layer and the external inviscid flow. It is well known that the boundary layer calculation requires the pressure variation along the edge, as given by the external inviscid flow. In the same time, since the displacement thickness effect modifies the effective body shape, the boundary layer influences the inviscid flow and thus changes the pressure distribution. This coupled interaction becomes very strong near the separation point.

In the classical approach, the pressure variation at the edge is given as a polynomial function, disregarding this interactive displacement effect of the boundary layer. In this case, as shown by many authors, the boundary layer equation solution exhibits a strong singularity in the streamwise behaviour of the wall friction $\tau \simeq (du/dy)_w$ at the separation point. For example, in the case of the flow past a plane with a given adverse pressure gradient of the form $dp/dx = 1 + (x_s - x)$, Goldstein[10] finds that du/dy behaves like $(x_s - x)^{1/2}$ (Fig. 7a) and the flow upstream cannot be linked up with the flow downstream separation point.

As also shown by Goldstein, it is possible for a certain prescribed pressure distribution that the singularity not appear and $(du/dy)_{wall} = 0$ for all x (e.g., the Falkner-Skan case). Goldstein further posed the

question whether there might be other special pressure distributions for which the singularity does not appear. It seems that Meksyn gives an affirmative answer: using a different method of analysis, he shows that the singularity does not occur when approaching the separation point, if the streamwise pressure variation corresponds to the real interactive one, as experimentally determined. The behaviour of $(du/dy)_{wall}$ in such a case is illustrated in Fig. 7b (as determined by Sfeir[8] for the case of the flow in a corner), and we can see that such a variation, although difficult to be represented by series expansions, does not exhibit any singularity at the separation point. The fact that Meksyn obtains good results proves that the flow is very sensitive to the pressure distribution which can be considered reponsible for the occurrance of the singularity, when arbitrarily given instead of being allowed to float free as part of a viscous-inviscid interactive solution.

As also reported by Meksyn, he faced some difficulties in the computations not in the separation point but in an upstream station close to the separation point, perhaps in the region of the rapid variation of $(du/dy)_{wall}$ (Fig. 7b).

In the following Meksyn's method will be discussed in order to adapt it for the calculation of the basic flow parameters entering the disturbance equations.

3.2 Adaption of Meksyn's Method for Two-Dimensional Separation

If the wall is insulating the energy equation,(26) admits the solution $g_o \equiv 1$ corresponding to the adiabatic flow ($H_o = H_e = H_\infty$ = const.). The dynamic equation (25) simplifies, becoming

$$\frac{\partial^3 f_o}{\partial \eta^3} + f_o \frac{\partial^2 f_o}{\partial \eta^2} = \Lambda\left[1 - \left(\frac{\partial f_o}{\partial \eta}\right)^2\right] + 2\xi\left(\frac{\partial f_o}{\partial \eta}\frac{\partial^2 f_o}{\partial \eta \partial \xi} - \frac{\partial f_o}{\partial \xi}\frac{\partial^2 f_o}{\partial \eta^2}\right) \qquad (45)$$

which cuincides in form with the equation of invompressible laminar boundary layer studied by Meksyn[11]. A short outline of his method of solutioning in given in Appendix B.

In the following we will direct our attention to some aspects related to the use of Meksyn's method for the calculation of the coefficients entering the disturbance equations. As we can see by examining the equations (39), (40), (41), and (42), these coefficients contain f_o, $\partial f_o/\partial\xi$, $\partial f_o/\partial\eta$, $\partial^2 f_o/\partial\xi\partial\eta$, $\partial^2 f_o/\partial\eta^2$.

Using the formula (B-4), we can write the following relations

$$f_o = \int_0^\eta d\eta \int_0^\eta e^{-F} \phi d\eta = \eta\int_0^\eta e^{-F} \phi\, d\eta - \int_0^\eta e^{-F} \eta\phi d\eta = \eta I_o - I_1$$

$$\frac{\partial f_o}{\partial\xi} = \eta\int_0^\eta e^{-F}\left(\frac{\partial\phi}{\partial\xi} - \frac{\partial F}{\partial\xi}\phi\right)d\eta - \int_0^\eta e^{-F}\eta\left(\frac{\partial\phi}{\partial\xi} - \frac{\partial F}{\partial\xi}\phi\right)d\eta = \eta I_{o\xi} - I_{1\xi}$$

$$\frac{\partial^2 f_o}{\partial\xi\partial\eta} = \int_0^\eta e^{-F}\left(\frac{\partial\phi}{\partial\xi} - \frac{\partial F}{\partial\xi}\phi\right)d\eta = I_{o\xi} \tag{46}$$

$$\frac{\partial^2 f_o}{\partial\eta^2} = \frac{1}{\eta}\int_0^\eta e^{-F}\left(\phi + \eta\frac{\partial\phi}{\partial\eta} - \eta\phi f_o\right) d\eta = \frac{I_3}{\eta}$$

where

$$I_o = \frac{\partial f_o}{\partial\eta} = \int_0^\eta e^{-F}\phi\, d\eta \quad \text{(see also B-7)}$$

$$I_{o\xi} = \int_0^\eta e^{-F}\left(\frac{\partial\phi}{\partial\xi} - \frac{\partial F}{\partial\xi}\phi\right) d\eta \tag{47}$$

$$I_1 = \int_0^\eta e^{-F}\eta\phi\, d\eta$$

$$I_{1\xi} = \int_0^\eta e^{-F} \cdot \eta \left(\frac{\partial\phi}{\partial\xi} - \frac{\partial F}{\partial\xi}\phi \right) d\eta$$

$$I_3 = \int_0^\eta e^{-F} \left(\phi + \eta\frac{\partial\phi}{\partial\eta} - \eta\phi f_0\right) d\eta$$

and f_0, F, ϕ are given by (B-3) and (B-5).

As we can see, the calculation of f_0, $\partial f_0/\partial\eta$, $\partial f_0/\partial\xi$, $\partial^2 f_0/\partial\xi\partial\eta$, $\partial f_0{}^2/\partial\eta^2$ reduces to the calculation of integrals I_0, $I_{0\xi}$, I_1, $I_{1\xi}$, I_3, all being of the form

$$\int_0^\eta e^{-F(\xi,\eta)} \psi(\xi,\eta)\, d\eta \tag{48}$$

where ψ is a slowly varying function which can be expanded in powers of η (the coefficients being functions of ξ)

$$\psi = \sum_{n=0}^{\infty} c_n \eta^n \tag{49}$$

To solve the integral (48) we use the method of "steepest descent" due to Debye[16] and widely applied by Meksyn to calculate similar integrals (see also Appendix B). This method is based on the idea that the value of the integral (48) comes mainly from the vicinity of the stationary point of F, which in our case is $\eta = 0$ (because $\partial F/\partial\eta = 0$ at $\eta = 0$).

The integral (48) is expressed in terms of incomplete Gamma functions $\Gamma(\frac{n+1}{4}, \tau)$

$$\int_0^\eta e^{-\tau} \psi\, d\eta = \lim_{\varepsilon\to 1} \sum_{n=0}^{\infty} d_n\, \Gamma\left(\frac{n+1}{4}, \tau\right) \varepsilon^{n+1} \tag{50}$$

where

$$\tau = F(\xi,\eta) = \eta^4 \sum_{n=4}^{\infty} \frac{a_{n-1}}{n!} \eta^{n-4} \tag{51}$$

and d_n's are functions of a_n (see Appendix B) and c_n, being given in Appendix C.

To improve the convergence of the series (50), we may use Euler's

transformation which yields

$$\int_0^\eta e^{-\tau} \psi \, d\eta = \sum_{n=0}^{\infty} p_n \, 2^{-(n+1)} \tag{52}$$

where

$$p_o = d_o \, \Gamma(1/4, \tau), \; . \; . \; . \; , \; p_n = d_o \, \Gamma(1/4, \tau) + \binom{n}{1} d_1 \, \Gamma(1/2, \tau)$$

$$+ \binom{n}{2} d_2 \, \Gamma(3/4, \tau) + . \; . \; . + d_n \, \Gamma\left(\frac{n+1}{n}, \tau\right)$$

$$\binom{n}{m} = \frac{n\,(n-1) \cdot \cdot \cdot (n-m+1)}{m!}$$

We note that the expression (51) relating τ and η can be used only for $\eta < 1$. But in the same time, we can use the following relation

$$\tau = \int_0^\eta d\eta \int_0^\eta d\eta \int_0^\eta e^{-F} \phi \, d\eta = \frac{\eta^2}{2} I_o - \eta I_1 + \frac{1}{2} I_2 \tag{53}$$

where $I_2 = \int_0^\eta e^{-F} \eta^2 \phi \, d\eta$ is also of the form (48).

Because I_o, I_1, I_2 are calculated in terms of τ, the equation (53) gives η in respect with τ

$$\eta = \frac{I_1 + \sqrt{I_1^{\,2} + 2I_o(1/2\; I_2 - \tau)}}{I_o} \tag{54}$$

To find τ by using (54) when η is given, an iterative procedure must be used.

3.3 Procedure to Find the Separation Point Location ξ_S and the Values of $(da/d\xi)_S$ and $(d^2a/d\xi)_S$

As shown in Appendix B, $f_o(\xi,\eta)$ is expressed in terms of ξ, $\Lambda(\xi)$, and $a(\xi)$, where $\Lambda(\xi)$ is a given function representing the pressure streamwise

gradient while $a(\xi)$ must be determined from the condition $\partial f_0/\partial\eta = 1$ at $\eta = \infty$ $\left[\text{see (B-7')}\right]$.

The location of the separation point ξ_s can be determined from the condition (A-1'''), i.e., $a(\xi_s) = 0$. In this case the relation (B-7') gives

$$\lim_{\varepsilon\to 1}\Bigg[0\cdot\varepsilon + 1/4(24\Lambda)^{1/2}\,\Gamma(1/2)\,\varepsilon^2 + 0\cdot\varepsilon^3 + 0\cdot\varepsilon^4 + 0\cdot\varepsilon^5$$

$$+\left(\frac{24}{\Lambda}\right)^{1/2}\frac{(4-15\Lambda)\Lambda + 10\xi\frac{d\Lambda}{d\xi}}{56}\,\Gamma(3/2)\,\varepsilon^6 + 0\cdot\varepsilon^7$$

$$+\frac{24}{25}\frac{\xi^2}{\Lambda}\left(\frac{da}{d\xi}\right)\Gamma(2)\,\varepsilon^8 + \ldots\Bigg] = 1 \qquad (55)$$

As we can see the first seven terms do not contain $da/d\xi$, this appearing only in the 8th term.

On the other hand, by derivating the relation (B-7') in respect with ξ we obtain at the separation location

$$\lim_{\varepsilon\to 1}\Bigg\{\frac{1}{4}\left(\frac{24}{\Lambda}\right)^{1/4}\frac{da}{d\xi}\,\Gamma(1/4)\,\varepsilon + \frac{1}{8}\left(\frac{24}{\Lambda}\right)^{1/2}\frac{d\Lambda}{d\xi}\,\Gamma(1/2)\,\varepsilon^2 + 0\cdot\varepsilon^3$$

$$+\frac{8\xi}{5\Lambda}\left(\frac{da}{d\xi}\right)^2\Gamma(1)\,\varepsilon^4 + \left(\frac{24}{\Lambda}\right)^{1/4}\Bigg[\frac{(-141\,\Lambda^2 - 66\,\Lambda + 94\xi\,\frac{d\Lambda}{d\xi})}{112}\frac{da}{d\xi}$$

$$\Gamma(5/4)\,\varepsilon^5\Bigg] + \frac{d}{d\xi}\Bigg[\left(\frac{24}{\Lambda}\right)^{1/2}\frac{(4 - 15\Lambda)\Lambda + 10\xi\,\frac{d\Lambda}{d\xi}}{56}\Bigg]\Gamma(3/2)\,\varepsilon^6$$

$$-\frac{119}{3600}\left(\frac{24}{\Lambda}\right)^{3/4}\frac{\xi^2}{\Lambda}\left(\frac{da}{d\xi}\right)^3\Gamma(7/4)\,\varepsilon^7 + \Bigg[\frac{8\,\xi}{525\Lambda}\left(\frac{da}{d\xi}\right)^2(118\Lambda - 75\Lambda^2$$

$$-139\xi\,\frac{d\Lambda}{d\xi}) + \frac{72}{25}\frac{\xi^2}{\Lambda}\frac{da}{d\xi}\frac{d^2a}{d\xi^2}\Bigg]\Gamma(2)\,\varepsilon^8 + \ldots\Bigg\} = 0 \qquad (56)$$

We can see that the first seven terms do not contain $d^2a/d\xi^2$ which appears

only in the eight term.

To find ξ_s, $(da/d\xi)_s$, and $(d^2a/d\xi^2)_s$ we can use the following procedure:

1. Using the first seven terms in Eq. (55) we find the separation point ξ_s

2. Using ξ_s determined above and considering only the first seven terms in (56), we find $(da/d\xi)_s$

3. Using $(da/d\xi)_s$ in the eight term of (55) we recalculate ξ_s

4. Using $(da/d\xi)_s$ and the new value for ξ_s we determine $(d^2a/d\xi^2)_s$ from (56).

As a general remark, to solve the equations (55) and (56) for the above situations an iterative procedure must be used, starting from a point $\xi = \xi_1$ and approaching the separation point $\xi = \xi_s$ where the equations (55) and (56) are satisfied. In all cases the Euler's transformation must be used to sum the series.

4 CONCLUDING DISCUSSION

The theoretical developments outlined in this report represent the background for the investigation of the spanwise disturbed two-dimensional flow near a separation location. The quasi-singular behaviour of the basic flow near the separation, requiring a correct, experimentally determined pressure distribution at the edge of the boundary layer, represents the main difficulty of the entire problem. The further steps towards a final solution of the problem are

1. Numerical computation of the basic flow parameters using the formulas outlined in Chapter 3. In this respect, the experimental results of Sfeir[8] for the two-dimensional separated flow in a corner can be chosen as a concrete situation. Due to the simplicity of pressure variation for this case, we can try to approximate it by analytical formulas of the form

$$p_e = p_\infty \quad \text{for } x < x_1$$

$$\frac{p_e}{p_\infty} = 1 + A\,[1 - e^{-\alpha(x-x_1)}]^n \quad \text{for } x_1 < x < x_2$$

where x_1 and x_2 delimit the range of rapid pressure variation while A, α, and n are parameters which can be adjusted in order to obtain a good approximation for the experimental curve.

2. Analytical and numerical study of the eigenvalue problem in order to establish the range of $\overline{\alpha}$ for which the disturbance equations admit non-trivial solutions.Taking into account the complexity of the disturbance equations a numerical computation seems to be the only possibility. The solution for the basic flow outlined in Chapter 3 suits only for the

adiabatic case. An analytical treatment can be tried only for an incompressible case.

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APPENDIX A

APPROXIMATE EXPRESSIONS FOR BOUNDARY LAYER INCLINATION & CURVATURE AT SEPARATION

We consider a steady two-dimensional high Reynolds number boundary layer flow without surface mass transfer and examine by appropriate limiting processes the behavior near the surface at the separation point $x = x_s$ $[(\partial u/\partial y)_{w,s} = 0]$.

Consider first the streamline inclination of the flow. Approaching the surface at any arbitrary $x < x_s$, a direct calculation of the streamline slope $\lim\limits_{y\to 0} (v/u)$ is indeterminate since both u and v vanish at $y = 0$; however, application of L'Hopital's rule plus the use of the continuity equation resolves this difficulty, giving

$$\lim_{y\to 0} \frac{u}{v} = \lim_{y\to 0} \frac{\partial v/\partial y}{\partial u/\partial y}$$

$$= \lim_{y\to 0} \left[\frac{- \rho^{-1} \dfrac{\partial(\rho u)}{\partial x} - v \dfrac{\partial \rho}{\partial y}}{\partial u/\partial y} \right] \tag{A-1}$$

which always vanishes (as it physically should) at the surface $u = v = \partial u/\partial x = 0$ for all $x < x_s$ where $(\partial u/\partial y)_w \neq 0$. Approaching the separation point where $(\partial u/\partial y)_w \to 0$, however, the right side of A-1 becomes indeterminate so we must again apply L'Hopital to obtain

$$\lim_{y\to 0} (v/u)_{x_s} = \lim_{\substack{y\to 0 \\ x\to x_s}} \left\{ \frac{- \dfrac{\partial}{\partial y} \left[\dfrac{1}{\rho} \dfrac{\partial(\rho u)}{\partial x} \right] - \dfrac{\partial}{\partial y} \left(v \dfrac{\partial \rho}{\partial y} \right)}{\dfrac{\partial}{\partial y} \left(\dfrac{\partial u}{\partial y} \right)} \right\}$$

$$= \lim_{\substack{y\to 0 \\ x\to x_s}} \left\{ \frac{\dfrac{\partial \rho/\partial y}{\rho^2} \dfrac{\partial(\rho u)}{\partial x} - \dfrac{1}{\rho} \dfrac{\partial}{\partial x} \left[\dfrac{\partial(\rho u)}{\partial y} \right] - \dfrac{\partial V}{\partial y} \dfrac{\partial \rho}{\partial y} - V \dfrac{\partial^2 \rho}{\partial y^2}}{\dfrac{\partial}{\partial y} \left(\dfrac{\partial u}{\partial y} \right)} \right\}$$

$$\equiv -\left[\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right) \Big/ \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)\right]_{wall, x_s} \tag{A-2}$$

which holds for an arbitrary streamwise pressure gradient, degree of compressibility and turbulent eddy viscosity model. But for any such flow we can write $\tau = (\mu + \mu_T)\ (\partial u/\partial y)$ while approaching the wall $\partial\tau/\partial y = \partial p_w/\partial x$; using these in (A-2) and assuming that any turbulence damps out at the wall ($\mu_{Tw} = 0$), we obtain

$$\lim_{y\to 0}\left(\frac{v}{u}\right)_{x_s} = \frac{-\ (\partial\tau_w/\partial x)_s}{(\partial p_w/\partial x)_s} \tag{A-3}$$

a result again valid for a perfectly general flow. Equation A-3 defines the slope of the separation streamline leaving the surface (Fig. 4).

Now consider the average curvature of the flow passing above this separation streamline. Assuming it is not large, we can write the associated radius of curvature to good approximation as

$$R_{AV}^{-1} \simeq -\frac{\partial\,(v/u)}{\partial\,x} = -\frac{\partial\,v/\partial\,x}{u} + \frac{v}{u}\frac{\partial\,u/\partial\,x}{u} \tag{A-4}$$

Approaching the surface (which is presumed flat) upstream of the separation point, this expression is indeterminate; then once more using L'Hopital gives

$$R_{AV\ y\to 0}^{-1} = \lim_{y\to 0}\left[\frac{-\frac{\partial}{\partial x}\left(\frac{\partial v}{\partial y}\right) + \frac{v}{u}\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right) + \frac{\partial y}{\partial x}\frac{\partial}{\partial y}\left(\frac{v}{u}\right)}{\partial u/\partial y}\right]$$

$$= 0 \qquad\qquad x < x_s$$

$$= \lim_{y\to 0}\left[\frac{\frac{v}{u}\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right)}{\partial u/\partial y}\right] = \frac{0}{0} \qquad \text{at } x = x_s$$

Resolving the latter indeterminate form, we get

$$R_{AV,s}^{-1} = \lim_{\substack{y \to 0 \\ x \to x_s}} \left[\frac{\frac{\partial^2}{\partial x^2}\left(\frac{\partial u}{\partial y}\right) + \left(\frac{v}{u}\right)_s \frac{\partial}{\partial x}\left(\frac{\partial^2 u}{\partial y^2}\right)}{\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)} \right]$$

which upon differentiations of the formulae $\tau = (\mu + \mu_T)(\partial u/\partial y)$ and $\partial\tau/\partial y = \partial p/\partial x$ and assuming $(\partial\mu_T/\partial y)_{w,s} \simeq 0$ ultimately yields the following expression:

$$R_{AV,s}^{-1} = \frac{\left(\frac{\partial^2 \tau_w}{\partial x^2}\right)_s + \left(\frac{\partial\tau_w/\partial x}{\partial p_w/\partial x}\right)\left[\frac{\partial^2 p_w}{\partial x^2} - \left(\frac{\partial\mu}{\partial y}\right)_{w,s} \frac{\partial\tau_w}{\partial x}\right] - \frac{3}{\mu_w}\left(\frac{\partial\mu_w}{\partial x}\right)_s\left(\frac{\partial\tau_w}{\partial x}\right)_s}{\frac{\partial p_w}{\partial x}} \quad \text{(A-5)}$$

Since in most applications $\partial^2 p_w/\partial x^2$ and $\partial\mu_w/\partial x \simeq \partial T_w/\partial x$ are very small if not in fact zero at x_s while the corresponding heat transfer and hence $(\partial\mu/\partial y)_s$ is also usually small, it is sufficient to neglect all but the first term in the numerator and get

$$R_{AV,s} \simeq \frac{(\partial p_w/\partial x)_s}{(\partial^2 \tau_w/\partial x^2)_s} \quad \text{(A-6)}$$

We observe from (A-6) that $R_{AV,s}$ is directly proportional to the curvature of the $\tau_w(x)$ plot and that it also decreases as the separation angle increases (i.e., in agreement with physical expectation, the curvature vanishes as this angle $\to$ 0).

APPENDIX B

MEKSYN'S SOLUTION FOR THE TWO-DIMENSIONAL FLOW IN THE BOUNDARY LAYER NEAR SEPARATION [11]

The problem is to find the solution of the equation (39), the boundary conditions being

$$f_o = \frac{\partial f_o}{\partial \eta} = 0 \text{ at } \eta = 0 \qquad (B\text{-}1')$$

$$\frac{\partial f_o}{\partial \eta} = 1 \text{ at } \eta = \infty \qquad (B\text{-}1'')$$

At the separation point there is an additional condition

$$\frac{\partial^2 f_o}{\partial \eta^2} = 0 \text{ at } \eta = 0 \qquad (B\text{-}1''')$$

As shown by Meksyn, in the region close to the wall ($\eta < 1$), $f_o(\xi,\eta)$ can be expanded in powers of η, the coefficients being functions of ξ, $\Lambda(\xi)$, and $a(\xi) = (\partial^2 f_o/\partial\eta^2)_{\eta=0}$

$$f_o(\xi,\eta) = \sum_{n=2}^{\infty} \frac{a_n \eta^n}{n!} \qquad (B\text{-}2)$$

In the neighbourhood of the separation point Meksyn gives the following expressions for a_n coefficients:

$$\begin{aligned}
&a_2 = 0 \quad [\text{hence } f_o(\xi,\eta) \text{ starts with } \eta^3, \text{ see } (B\text{-}1''')];\\
&a_3 = \Lambda; \quad a_4 = 0; \quad a_5 = 2\xi\, a\, (da/d\xi);\\
&a_6 = -(4 + 6\Lambda)\Lambda a + 4\xi a\, (d\Lambda/d\xi);\\
&a_7 = -(4 + 6\Lambda)\Lambda^2 + 4\xi\Lambda\, (d\Lambda/d\xi);\\
&a_8 = -4\xi^2 a\, (da/d\xi)^2;\\
&a_9 = -2(10 + 42\Lambda)\xi\Lambda a \frac{da}{d\xi} + 2\xi^2\left[-26a \frac{d\Lambda}{d\xi}\frac{da}{d\xi} + 18\left(\frac{da}{d\xi}\right)^2 + 18\Lambda a\frac{d^2 a}{d\xi^2}\right]
\end{aligned} \qquad (B\text{-}3)$$

The solution of the equation (B-1) can also be expressed in the form

$$\frac{\partial^2 f_o}{\partial y^2} = e^{-F}\,\phi(\xi,\eta) \quad , \quad F = \int_0^\eta f_o \, d\eta \tag{B-4}$$

where $\phi(\xi,\eta)$ is a function whose expansion in powers of η near the wall is given by

$$\phi(\xi,\eta) = \sum_{n=0}^{\infty} \frac{b_n \eta^n}{n!} \tag{B-5}$$

In the region of separation b_n's have the following expressions

$$\begin{aligned} & b_o = a; \quad b_1 = \Lambda; \quad b_2 = 0; \quad b_3 = 2\xi a \frac{da}{d\xi} ; \\ & b_4 = -3(1 + 2\Lambda)\Lambda a + 4\xi \frac{d\Lambda}{d\xi} a ; \\ & b_5 = (1 - 6\Lambda)\Lambda^2 + 4\xi\Lambda \frac{d\Lambda}{d\xi} ; \\ & b_6 = -4\xi^2 a \left(\frac{da}{d\xi}\right)^2 \end{aligned} \tag{B-6}$$

Meksyn demonstrates that the relation (B-4) also describes the asymptotic behaviour of $(\partial^2 f_o/\partial\eta^2)$ far from the wall, which shows that $\phi(\xi,\eta)$ must be considered a slowly varying function.

Although the expansion (B-5) of ϕ is divergent for $\eta \geq 1$, the relation (A-4) can still be used to approximate

$$\frac{\partial f_o}{\partial \eta} = \int_0^\eta e^{-F}\,\phi(\xi,\eta)\, d\eta$$

because F has $\eta = 0$ as a stationary point, and as a consequence, the value of the integral comes mainly from the points near the wall where F changes very rapidly.

Using this idea, Meksyn gives the following expression for $\partial f_o/\partial\eta$ in

terms of $\tau = F(\eta)$ and incomplete Gamma functions $\Gamma(\frac{n+1}{4}, \tau)$

$$\frac{\partial f_o}{\partial \eta} = \int_0^{\eta} e^{-F} \phi \, d\eta = \lim_{\varepsilon \to 1} \sum_{n=0}^{\infty} d_n \, \Gamma(\frac{n+1}{4}, \tau) \, \varepsilon^{n+1} \tag{B-7}$$

where d_n's have the following expressions near the separation point

$$d_o = \frac{1}{4}\left(\frac{24}{\Lambda}\right)^{1/4} a \; ; \quad d_1 = \frac{1}{4} (24\Lambda)^{1/2} ; \quad d_2 = 0 :$$

$$d_3 = \frac{8\xi a \frac{da}{d\xi}}{5\Lambda} \; ;$$

$$d_4 = \left(\frac{24}{\Lambda}\right)^{1/4} \frac{(-14\Lambda^2 - 66\Lambda + 94\xi(d\Lambda/d\xi)\, a}{112} \; ;$$

$$d_5 = \left(\frac{24}{\Lambda}\right)^{1/2} \frac{(4 - 15\Lambda)\Lambda \; + 10\xi\Lambda'}{56} \; ; \tag{B-8}$$

$$d_6 = - \frac{119}{3600} \left(\frac{24}{\Lambda}\right)^{3/4} \frac{\xi^2 a (da/d\xi)^2}{\Lambda}$$

$$d_7 = \frac{8\xi a \; (da/d\xi)}{525\Lambda} (118\Lambda \; - 75\Lambda^2 - 139\xi \frac{d\Lambda}{d\xi}) + \frac{24}{25} \frac{\xi^2 \left[\frac{da}{d\xi}^2 + a \frac{d^2 a}{d\xi^2}\right]}{\Lambda}$$

The unknown function $a(\xi)$ entering the above relations is found by using the boundary condition (B-1'') which furnishes the following differential equation

$$\frac{\partial f_o}{\partial \eta} = \lim_{\varepsilon \to 1} \sum_{n=0}^{\infty} d_n \, \Gamma(\frac{n+1}{4}) \, \varepsilon^{n+1} = 1 \tag{B-7'}$$

where $\Gamma(\frac{n+1}{4})$ are complete Gamma functions.

To improve the convergence of the above series when $\varepsilon \to 1$, use must be made of the well-known Euler's transformation [see also (52)], this giving

$$\frac{\partial f_o}{\partial \eta} = \lim_{\varepsilon \to 1/2} \sum_{n=0}^{\infty} p_n \, \varepsilon_1^{n+1} = 1 \tag{B-7''}$$

where

$$p_o = d_o \, \Gamma(1/4), \; . \; . \; . \; , \; p_n = d_o \, \Gamma(1/4) + \binom{n}{1} d_1 \, \Gamma(1/2) + \binom{n}{2} d_2 \, \Gamma(3/4)$$

$$+ \; . \; . \; . \; + \binom{n}{n} d_n \, \Gamma\left(\frac{n+1}{4} \right)$$

$$\binom{n}{p} = \frac{n\,(n-1) \cdot \cdot \cdot (n-p+1)}{p!}$$

Usually, the first six terms in (B-7'') are sufficient.

APPENDIX C

d_n COEFFICIENTS ENTERING THE RELATION (50)

$$d_o = \frac{1}{4}\left(\frac{a_3}{4!}\right)^{-1/4} c_o$$

$$d_1 = \frac{1}{4}\left(\frac{a_3}{4!}\right)^{-1/2} c_1$$

$$d_2 = \frac{1}{4}\left(\frac{a_3}{4!}\right)^{-3/4} \left(c_2 - \frac{1}{40}\frac{a_5}{a_3} c_o\right)$$

$$d_3 = \frac{1}{4}\left(\frac{a_3}{4!}\right)^{-1} \left(c_3 - \frac{1}{30}\frac{a_5}{a_3} c_1 - \frac{1}{210}\frac{a_6}{a_3} c_o\right)$$

$$d_4 = \frac{1}{4}\left(\frac{a_3}{4!}\right)^{-5/4} \left[c_4 - \frac{1}{24}\frac{a_5}{a_6} c_2 - \frac{1}{168}\frac{a_6}{a_3} c_1 + \left(\frac{1}{640}\frac{a_5^{2}}{a_3^{2}} - \frac{1}{1344}\frac{a_7}{a_3}\right) c_o\right]$$

$$d_5 = \frac{1}{4}\left(\frac{a_3}{4!}\right)^{-3/2} \left[c_5 - \frac{1}{20}\frac{a_5}{a_3} c_3 - \frac{1}{140}\frac{a_6}{a_3} c_2 + \left(\frac{1}{480}\frac{a_5^{2}}{a_3^{2}} - \frac{1}{1120}\frac{a_7}{a_3}\right) c_1\right.$$
$$\left. + \left(\frac{1}{1680}\frac{a_5 a_6}{a_3^{2}} - \frac{1}{10080}\frac{a_8}{a_3}\right) c_o\right]$$

$$d_6 = \frac{1}{4}\left(\frac{a_3}{4!}\right)^{-7/4} \left[c_6 - \frac{7}{120}\frac{a_5}{a_3} c_4 - \frac{1}{120}\frac{a_6}{a_3} c_3 + \left(\frac{77}{2(120)^2}\frac{a_5^{2}}{a_3^{2}} - \frac{1}{960}\frac{a_7}{a_3}\right) c_2\right.$$
$$+ \left(\frac{11}{(120)^2}\frac{a_5 a_6}{a_3^{2}} - \frac{1}{960}\frac{a_8}{a_3}\right) c_1 + \left(- \frac{77}{48\,(120)^2}\frac{a_5^{3}}{a_3^{3}} + \frac{11}{14(120)^2}\frac{a_6^{2}}{a_3^{2}}\right.$$
$$\left.\left. + \frac{11}{8(120)^2}\frac{a_5 a_7}{a_3^{2}} - \frac{1}{6(120)^2}\frac{a_9}{a_3}\right) c_o\right]$$

$$d_7 = \frac{1}{4}\left(\frac{a_3}{4!}\right)^{-2}\Bigg[c_7 - \frac{1}{15}\frac{a_5}{a_3}c_5 - \frac{1}{105}\frac{a_6}{a_3}c_4 + \left(\frac{1}{90}\frac{a_5^{\,2}}{a_3^{\,2}} - \frac{1}{840}\frac{a_7}{a_3}\right)c_3$$

$$+\left(\frac{1}{1050}\frac{a_5a_6}{a_3^{\,2}} - \frac{1}{840}\frac{a_8}{a_3}\right)c_2 + \left(-\frac{1}{6750}\frac{a_5^{\,3}}{a_3^{\,3}} + \frac{1}{2450}\left(\frac{a_6}{a_3}\right)^2 + \frac{1}{8400}\frac{a_5a_7}{a_3^{\,2}}\right.$$

$$\left.-\frac{1}{85050}\frac{a_9}{a_3}\right)c_1 + \left(-\frac{1}{15750}\frac{a_5^{\,2}a_6}{a_3^{\,2}} + \frac{1}{58800}\frac{a_6a_7}{a_3^{\,2}} + \frac{1}{85050}\frac{a_5a_8}{a_3^{\,2}}\right.$$

$$\left.-\frac{1}{831600}\frac{a_{10}}{a_3}\right)c_o\Bigg]$$

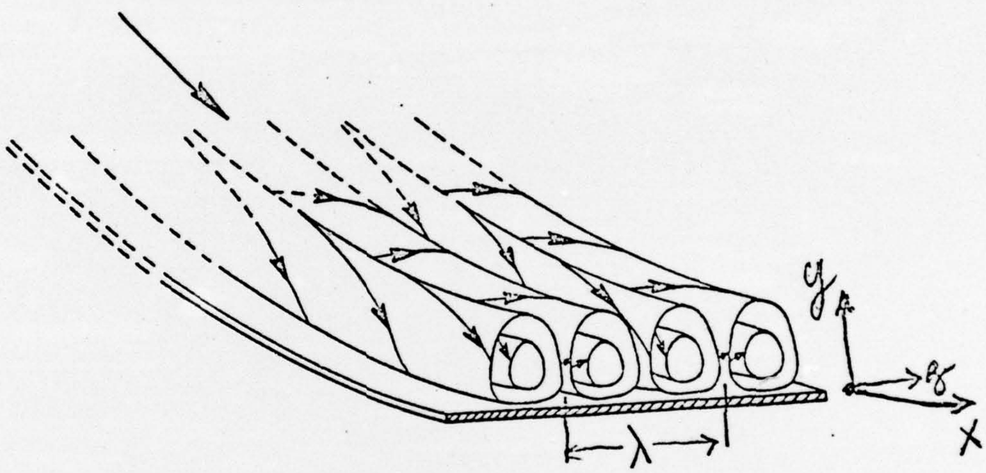
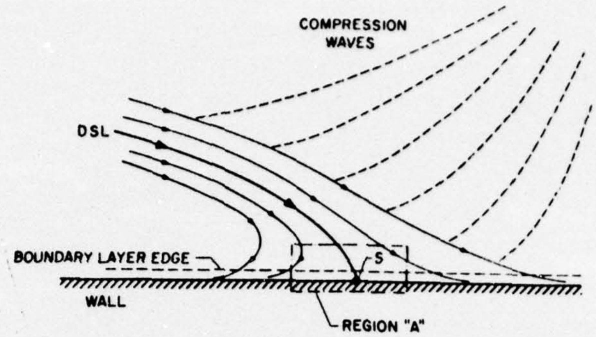


Fig. 1 Görtler's Streamwise Vortices



Reattachment Flow Configuration (Schematic)

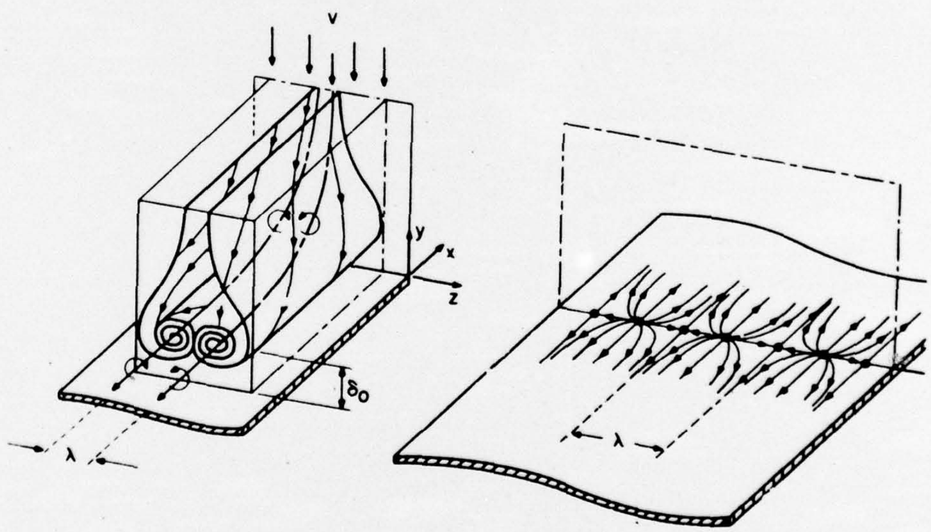


Fig. 2 Spanwise-Periodic Disturbances in Reattaching Flows (Inger)

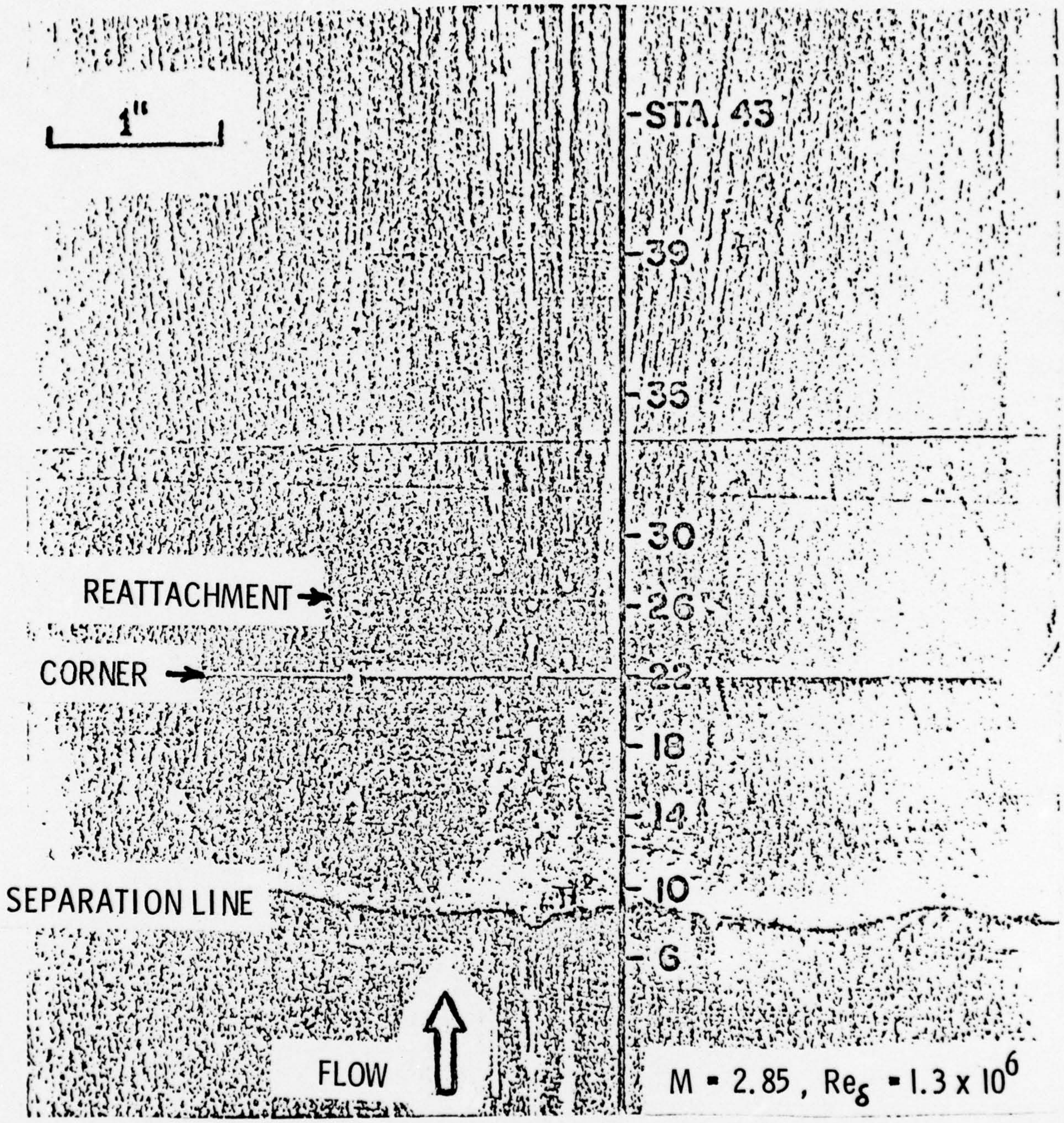
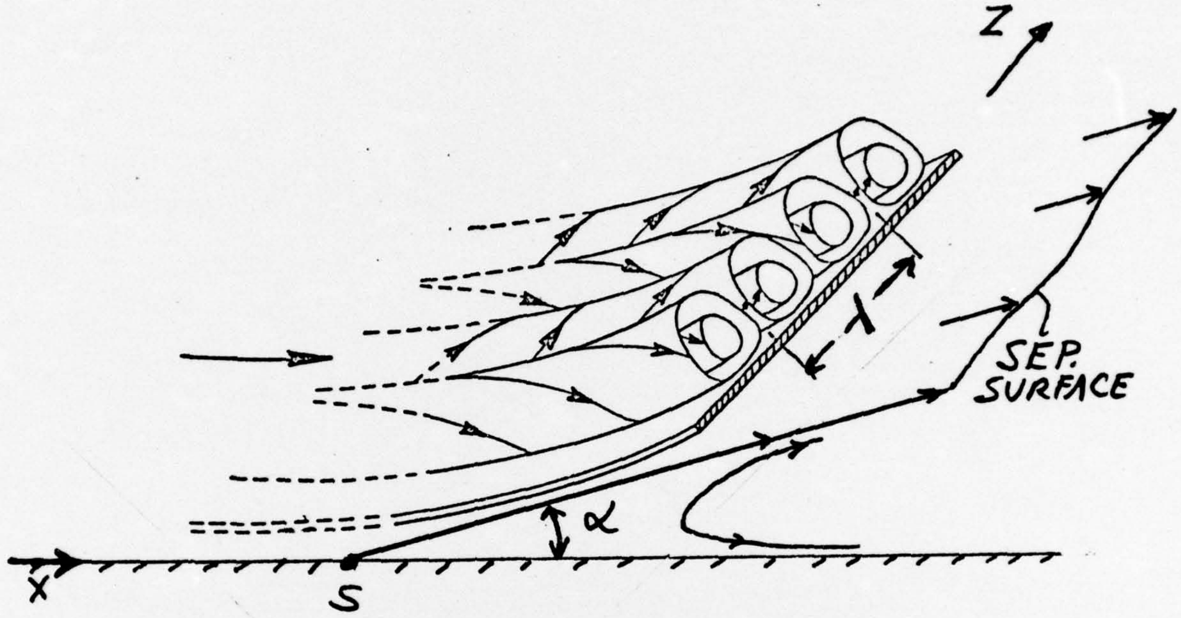


Fig. 3 Detailed Surface Flow Pattern Near Ramp-Induced Separation Line (Settles, et al)

λ = SPANWISE WAVELENGTH

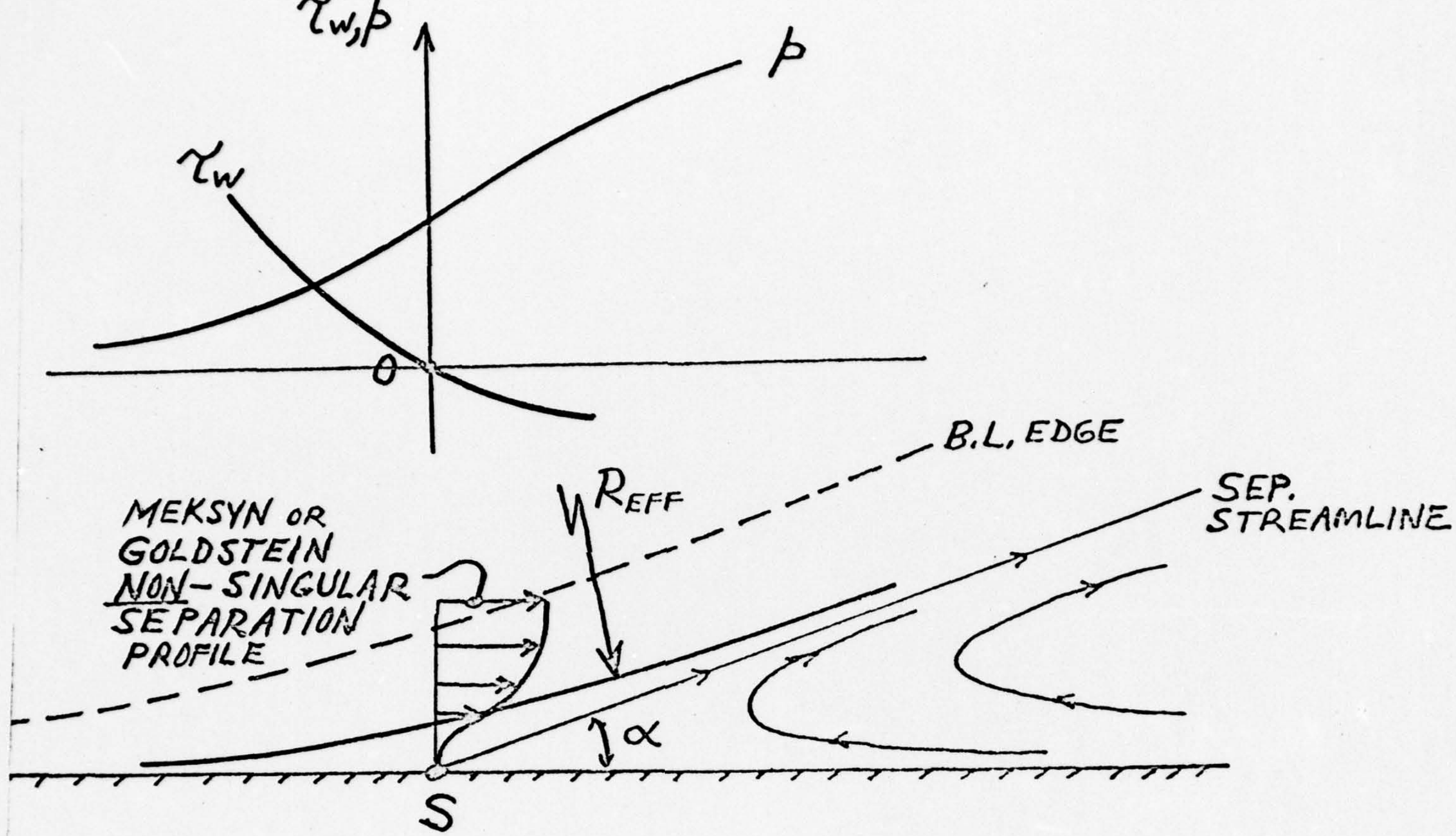


$$u = u_0(x_s, y) + u_1(y)\cos(2\pi z/\lambda) + O(x - x_s)^2$$

$$v = v_0(x_s, y) + v_1(y)\cos(2\pi z/\lambda) + \cdots$$

$$w = w' = w_1(y)\sin(2\pi z/\lambda) + \cdots$$

Fig. 4 Schematic of Assumed 3-D Disturbance Field in Separating Flow Region



$$\tan \alpha \simeq -(\partial \tau_w/\partial x)_s \Big/ (\partial p/\partial x)_s$$

$$R_{EFF} \simeq \frac{(\partial \tau_w/\partial x)_s}{(\partial^2 \tau_w/\partial x^2)_s} \cot \alpha = \frac{(\partial p/\partial x)_s}{(\partial^2 \tau_w/\partial x^2)_s}$$

Fig. 5 Schematic Illustration of Separation Region Inclination and Curvature Estimation

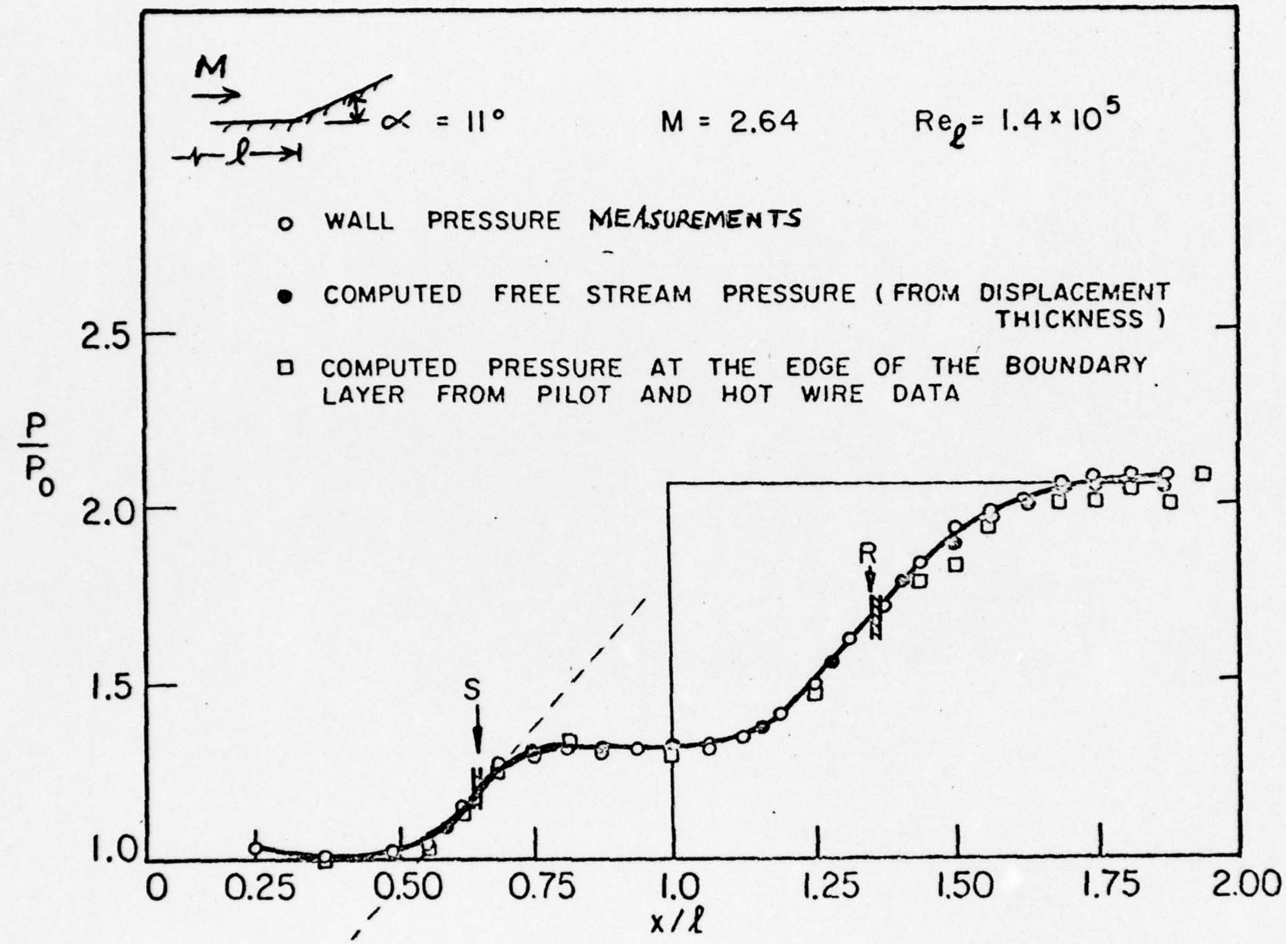


Fig. 6 Pressure Distribution Along a Ramp-Induced Separation Region (Steir)

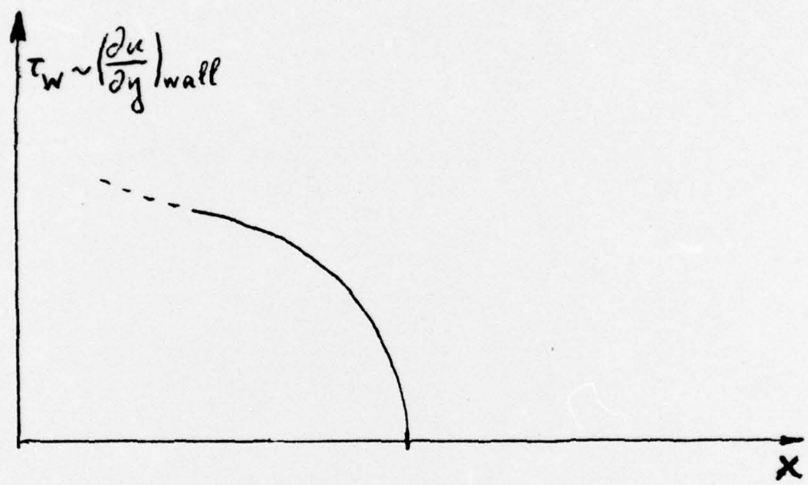


Fig. 7a Goldstein's Singular Shear Stress Distribution

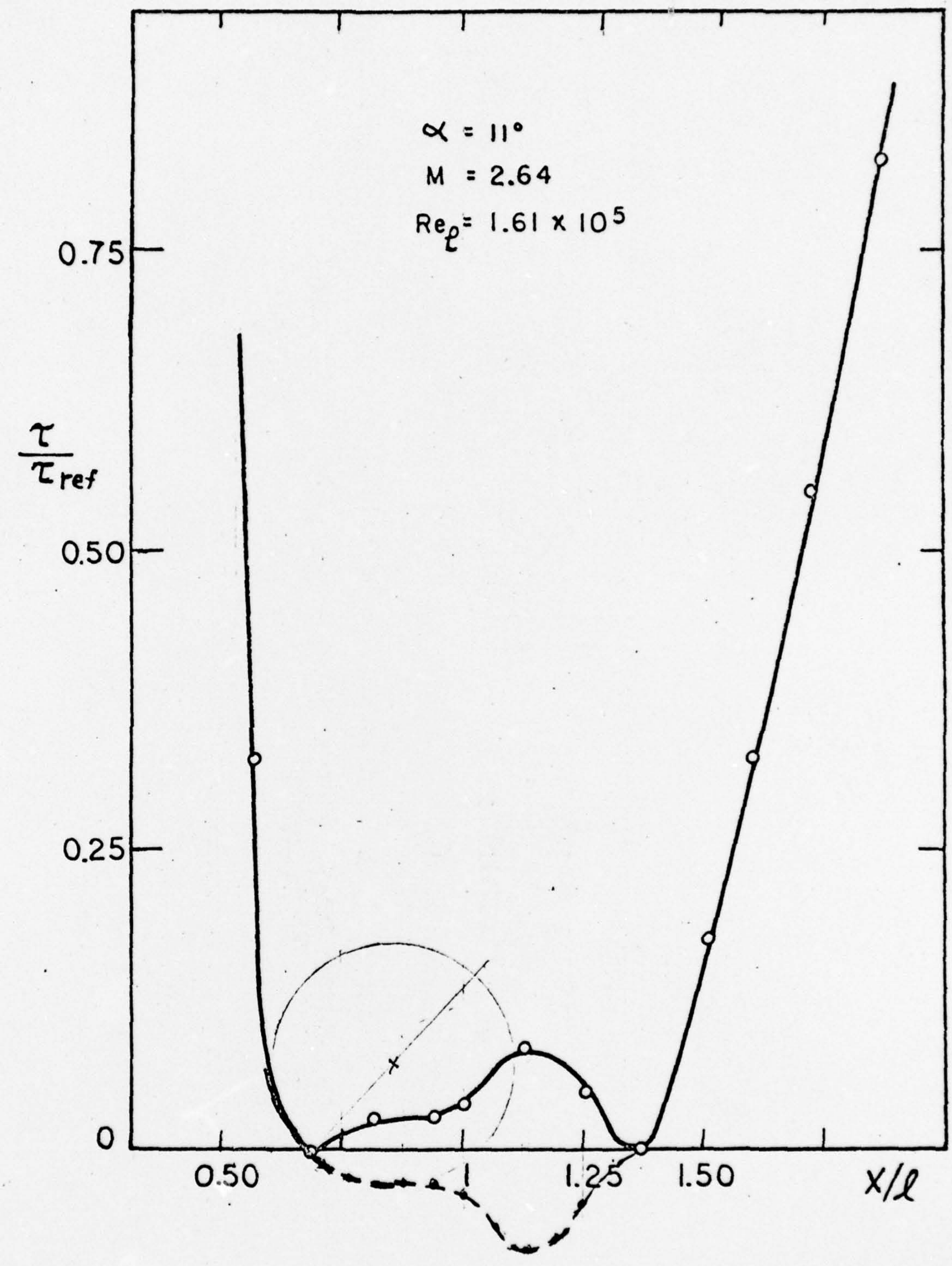


Fig. 7b Wall Shear Stress Distribution Along a Ramp-Induced Separation Region (Steir)

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LAMINAR BOUNDARY LAYER SEPARATION

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The results of a preliminary investigation on the stability of a nominally-two-dimensional laminar boundary layer flow approaching the separation point with respect to 3-D periodic spanwise disturbances, under the hypothesis that a steady state is reached, are presented. The basic equations of the disturbances are examined with an assumed form of the 3-D perturbations, resulting in a system of ordinary differential equations. Together with the boundary conditions requiring the disturbances to be zero at the wall and to vanish asymptotically at infinity, a two-point eigenvalue problem was formulated. Meksyn's method was adapted for→

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the calculation of the basic two-dimensional flow parameters entering the equations in the region near the separation point. This method requires that the experimentally determined streamwise variation of the pressure at the edge of the boundary layer be known a priori.